MODERN ALGEBRA

MATH 4030/8036

Course Description:

Algebra is the study of mathematical manipulations that preserve something (like equality –when solving equations). The areas in which Algebra finds application are quite diverse, from Ancient Greek Geometry through to Modern Information Protection and Security (error correcting codes, data compression, and cryptography). This course begins with topics that should be familiar (such as ruler-and-compass constructions, and modular arithmetic) and builds upon this foundation through polynomial rings up to finite fields and basic group theory. **3 credits**

Prerequisites:

Math 2230 with a C- or better or Math 2030 with a C- or better

Overview of content and purpose of the course:

The purpose of this course is to deepen student understanding of some familiar algebraic concepts, broaden this conceptual base, and expand student understanding of the applicability of Algebra. As a consequence, the course will start with familiar topics, then rapidly move them into new environments. This will necessitate the development of a language to describe the common features among these various environments, and this is the language of modern algebra.

Major Topics:

1) The Early History
   a. The Breakthrough

2) Complex Numbers
   a. Rational Functions of Complex Numbers
   b. Complex Roots
   c. Solvability by Radicals I
   d. Ruler-and-Compass Constructibility
   e. Orders of Roots of Unity

3) Solutions of Equations
   a. The Cubic Formula
   b. Solvability by Radicals II

4) Modular Arithmetic
   a. Modular Addition, Subtraction, and Multiplication
   b. The Euclidean Algorithm and Modular Inverses

5) The Binomial Theorem and Modular Powers
   a. The Binomial Theorem
   b. Fermat's Theorem and Modular Exponents

6) Polynomials over a Field
   a. Fields and Their Polynomials
   b. The Factorization of Polynomials
   c. The Euclidean Algorithm for Polynomials
   d. Elementary Symmetric Polynomials
   e. Lagrange's Solution of the Quartic Equation

7) Galois Fields
   a. Galois's Construction of His Fields
   b. The Galois Polynomial
   c. The Primitive Element Theorem
   d. On the Variety of Galois Fields

8) Permutations
   a. Permuting the Variables of a Function I
   b. Permutations
   c. Permuting the Variables of a Function II
   d. The Parity of a Permutation

9) Groups
   a. Permutation Groups
   b. Abstract Groups
   c. Isomorphisms of Groups and Orders of Elements
   d. Subgroups and Their Orders
   e. Cyclic Groups and Subgroups
   f. Cayley's Theorem

10) Quotient Groups and Their Uses
   a. Quotient Groups

Textbook:

Stahl, Saul. *Introductory Modern Algebra: A Historical Approach, 2nd ed*. Hoboken: John Wiley & Sons, 2013.

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